

# An Extended Relaxation Technique for Modeling Unsteady Flows in Channel Networks using the NWS FLDWAV Model

J.M. Lewis<sup>1</sup>, D.L. Fread<sup>2</sup>, Ming Jin<sup>3</sup>

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<sup>1</sup>Research Hydrologist, <sup>2</sup>Director, Office of Hydrology, <sup>3</sup> Visiting Scientist, Hydrologic Research Laboratory, Office of Hydrology, NOAA, National Weather Service, 1325 East-West Hwy., Silver Spring, MD 20910.

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## **Abstract**

A recent enhancement to the FLDWAV model is the addition of an extended relaxation technique for modeling unsteady flows in a dendritic (tree-type) network of channels with tributaries of nth (any) order. The current relaxation technique is limited to first-order tributaries. By applying a particular numbering scheme to the tributaries and adding a simple algorithm, the current relaxation algorithm is extended to river systems with nth-order tributaries while retaining its inherent computational efficiency and providing greater applicability.

## **Introduction**

The NWS FLDWAV model (Fread, 1985, 1993) is a generalized flood routing model which is based on the weighted, four-point, nonlinear, finite-difference solution of the one-dimensional equations of unsteady flow (Saint-Venant equations). FLDWAV combines the capabilities of the popular NWS DAMBRK and DWOPER models and has several enhancements. Some of the special features of the FLDWAV model include: flow routing through a system of interconnected waterways, an enhanced subcritical/supercritical mixed-flow solution algorithm, levee overtopping/floodplain interactions, automatic calibration of Manning roughness coefficients for historical floods, and selection of dynamic (implicit, explicit), diffusion, Muskingum-Cunge, and level-pool routing techniques throughout the river system. A recent enhancement to the FLDWAV model is the addition of an extended relaxation technique for modeling unsteady flows in a dendritic (tree-type) network of channels including tributaries of nth (any) order. By applying a particular numbering scheme to the

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<sup>1</sup>Research Hydrologist, <sup>2</sup>Director, Office of Hydrology, <sup>3</sup>Visiting Scientist, Hydrologic Research Laboratory, Office of Hydrology, NOAA, National Weather Service, 1325 East-West Hwy., Silver Spring, MD 20910.

tributaries and adding a simple algorithm, the current relaxation algorithm is extended to river systems with nth-order tributaries while retaining its inherent computational efficiency and providing greater applicability. This paper describes the extended relaxation technique and provides a numerical example of its application.

### **Model Structure**

The governing equations of the FLDWAV model are: the one-dimensional, unsteady flow equations originally derived by Saint-Venant; an assortment of internal boundary equations of flow through one or more flow control structures located along the main-stem river and/or its tributaries; and external boundary equations of known upstream/downstream discharges or water elevations which vary either with time or each other.

The expanded Saint-Venant equations of conservation of mass and momentum consist of the following (Fread, 1993):

$$\partial Q / \partial x + \partial s_c (A + A_o) / \partial t - q = 0 \quad (1)$$

$$\partial (s_m Q) / \partial t + \partial (\beta Q^2 / A) / \partial x + gA (\partial h / \partial x + S_f + S_e) + L + W_f \beta = 0 \quad (2)$$

in which  $Q$  is discharge (flow),  $A$  is wetted active cross-sectional area,  $A_o$  is wetted inactive off-channel (dead) storage area associated with topographical embayments or tributaries,  $B$  is the channel flow width,  $s_c$  and  $s_m$  are depth-dependent sinuosity coefficients for mass and momentum, respectively, that account for meander,  $\beta$  is the momentum coefficient for nonuniform velocity,  $q$  is lateral flow (inflow is positive, outflow is negative),  $t$  is time,  $x$  is distance measured along the mean flow-path of the floodplain,  $g$  is the gravitational acceleration constant,  $h$  is the water surface elevation,  $L$  is the momentum effect of lateral flows ( $L = -qv_x$  for lateral inflow where  $v_x$  is the lateral inflow velocity in the  $x$ -direction,  $L = -qQ/(2A)$  for seepage lateral outflows,  $L = -qQ/A$  for bulk lateral outflows such as flows over levees),  $S_f$  is the boundary friction slope,  $S_e$  is the slope due to local expansion-contraction (large eddy loss), and  $W_f$  is the wind term.

There may be various locations (internal boundaries) along the main-stem and/or tributaries where the flow is rapidly varied in space and the Saint-Venant equations are not applicable, e.g., dams, bridges/road-embankments, waterfalls, short steep rapids, weirs, etc. In lieu of Eqs. (1-2), various internal boundary equations as described by Fread (1985, 1993) may be used.

External boundary equations at the upstream and downstream extremities of the waterway must be specified to obtain solutions to the Saint-Venant equations. External boundary equations represent a specified time series of discharge (discharge hydrograph) or water elevation, as in the case of a lake level or estuarial tidal fluctuation. At the downstream extremity, the boundary equation can be an empirical

rating of  $h$  and  $Q$ , or a channel control, loop rating based on the Manning equation in which the dynamic energy slope is used.

Eqs. (1-2) are nonlinear, partial, differential equations which are solved by a weighted, four-point, nonlinear, implicit, finite-difference technique as described by Fread (1985). Substitution of appropriate simple algebraic approximations for the derivative and nonderivative terms in Eqs. (1-2) result in two nonlinear algebraic equations for each  $\Delta x$  reach between specified cross sections which, when combined with the external boundary equations and any necessary internal boundary equations, may be solved by an iterative quadratic solution technique (Newton-Raphson) along with an efficient, compact, quad-diagonal Gaussian elimination matrix solution technique. Initial conditions required at  $t=0$  are automatically obtained via a steady flow backwater solution. A river system consisting of a main-stem river and one or more principal tributaries is efficiently solved using an iterative relaxation method (Fread, 1973, 1985). If the river consists of bifurcations, such as islands and/or complex dendritic systems, with tributaries connected to tributaries, etc., a network solution technique is used (Fread, 1985), wherein three internal boundary equations conserve mass and momentum at each confluence. Solution of this system of algebraic equations requires another special sparse-matrix Gaussian elimination technique.

### **Extended Relaxation Technique**

During a time step, the relaxation algorithm solves the Saint-Venant equations first for the main river and then separately for each tributary of the first-order dendritic network. The tributary flow at each confluence with the main river is treated as lateral flow ( $q$ ) which is estimated when solving Eqs. (1-2) for the main river. Each tributary flow depends on its upstream boundary condition, lateral inflows along its reach, and the water elevation at the confluence (downstream boundary for the tributary) which is obtained during the simulation of the main river. Due to the interdependence of the flows in the main river and its tributaries, the following iterative or relaxation algorithm (Fread, 1973, 1985) is used:

$$q' = \alpha q + (1 - \alpha)q'' \quad (3)$$

in which  $q$  is the computed tributary flow at each confluence,  $q''$  is the previous estimate of  $q$ ,  $q'$  is the new estimate of  $q$ , and  $\alpha$  is a weighting factor ( $0 < \alpha \leq 1$ ). Convergence is attained when  $q$  is sufficiently close to  $q''$ , i.e.,  $|q - q''| < \epsilon_Q$ .

The acute angle ( $\omega_t$ ) that the tributary makes with the main river is a specified parameter. This enables the inclusion of the momentum effect of the tributary inflow via the term ( $L = -qv_t$ ) as used in Eq. (2). The velocity of the tributary inflow is given by:

$$v_x = (Q/A)_N \cos \omega_t \quad (4)$$

in which  $N$  denotes the last cross section of the tributary.

The current relaxation technique is limited to first-order tributaries. A dendritic system containing higher-order tributaries (Figure 1) may be modeled using this algorithm by redefining the main stem and treating the higher order tributaries as lateral flows. The FLDWAV model currently requires the user to supply the following information for each tributary:  $\omega$ ,  $\alpha$ ,  $\epsilon_0$ , and the cross-section location along the main river immediately upstream of tributary confluence (NJUN(j) where  $j=2,3,\dots,JN$  in which  $JN=1+\text{total number of tributaries}$ ). If, in addition to these parameters, the user also supplies the number of the river into which the tributary flows (MRV(j)), the relaxation algorithm may be used, without modification, to model river systems containing nth-order tributaries. The main river is always numbered 1 while the tributaries are numbered 2,3,...,JN.

The initial conditions, if not specified by the user, may be generated by assuming steady flow in the system and adding the inflow from each flow successively starting with the nth-order tributary and proceeding down to the main river. Backwater computations are done starting with the water elevation at the downstream boundary of the main river and proceeding to the upstream location on each river according to the order number (i.e., main stem, all 2nd-order tributaries, .... all nth-order tributaries). The computation order (IORDR(K)) of rivers needed for backwater computations is determined using the following algorithm:

```

K=1
IORDR(1)=1
for each river J
  for each river L
    if MRV(L)=J then
      set IORDR(K)=L
      K=K+1
    endif
  next L
next J

```

When computing the initial discharges, the computational order of rivers is the reverse of that needed for backwater computations. The current relaxation technique requires tributaries to be ordered from upstream to downstream. The new algorithm allows the flexibility of adding tributaries to the system in any order without affecting the numbering scheme of the original river system.

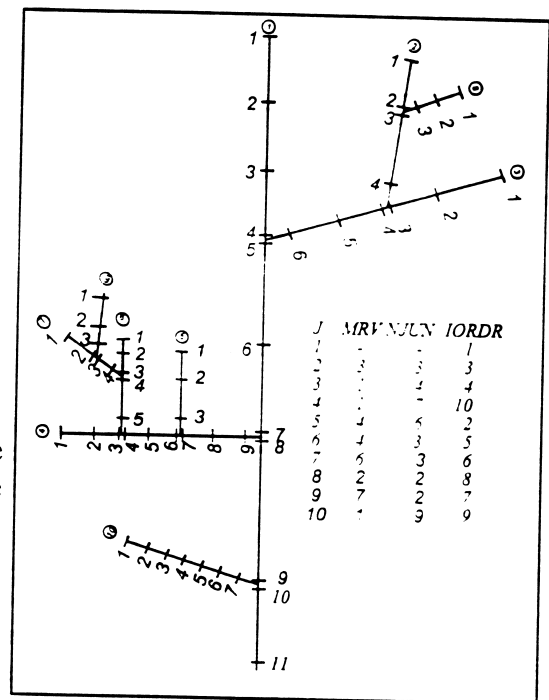


Figure 1. Dendritic River System

## Application

The FLDWAV model is applied to the river system depicted in Figure 2. The downstream boundary condition on river R1 is a stage-discharge relationship (Figure 3). The inflow hydrograph data and hydraulic information for each river are given in Tables 1 and 2, respectively. The initial conditions are steady flow throughout the system with a normal depth of 1.33 meters at the downstream end of the main river (R1). Backwater computations determine the initial water surface elevation throughout the system. Three scenarios are applied to this river system to determine the effect of modeling river R3 as a lateral inflow vs. a dynamic tributary with its bottom slope being mild ( $S_0=0.00568$ ), or flat ( $S_0=0.000095$ ). Discharge hydrographs are compared at a reference cross section located immediately below the confluence of R3 (6.21 km from the mouth of R2). Figure 4 shows the hydrographs at the reference section for each scenario. Figure 5 represents the inflow hydrograph in R3 and discharge hydrographs at the downstream cross section in the case of the mild and flat slope scenarios. When modeling river R3 as a lateral flow, a maximum discharge of 35.6 cms occurs in 60 hours at the reference section. When modeling river R3 as a dynamic tributary with a mild slope, a maximum discharge of 35.5 cms

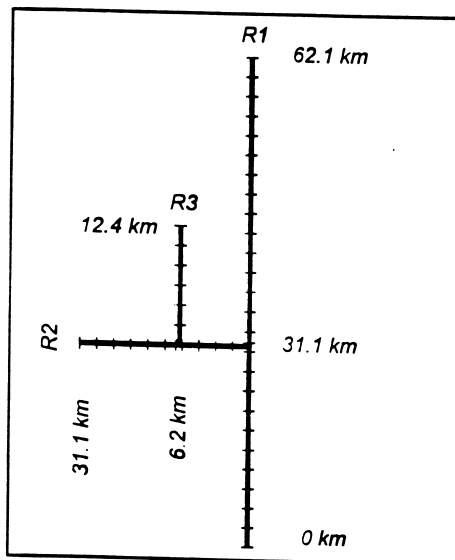


Figure 2. Schematic

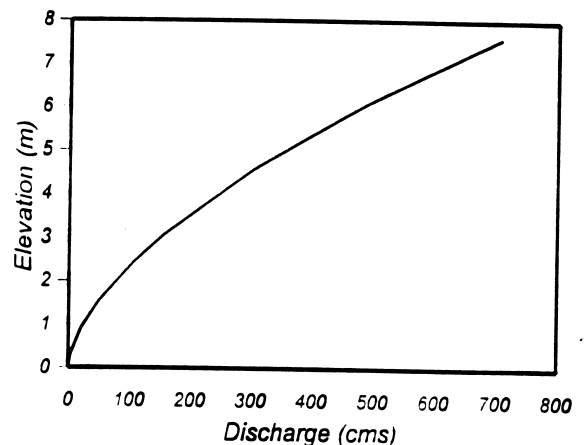


Figure 3. Stage-Discharge Relationship

Time (hrs)	0	12	60	108	200
River R1	28.3	28.3	283.2	28.3	28.3
River R2	8.5	8.5	8.5	8.5	8.5
River R3	2.8	2.8	28.3	2.8	2.8

Table 1. Inflow Hydrographs (cms)

River	Channel Width(m)	Bottom Slope	$\Delta x$ (km)	Manning n
R1	61	.000189	.62	.035
R2	9	.000189	.62	.035
R3	3	.000095	.06	.035
		.005681	3.1	.060

Table 2. Hydraulic Information

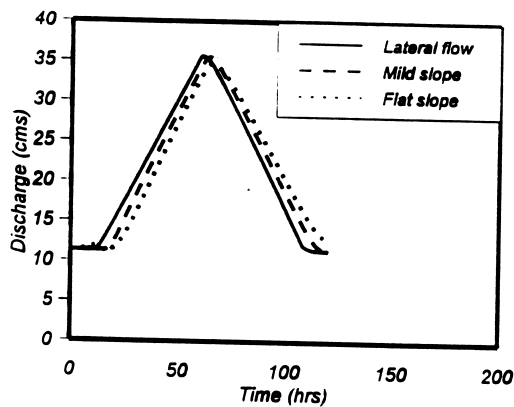


Figure 4. Discharge hydrographs at reference section.

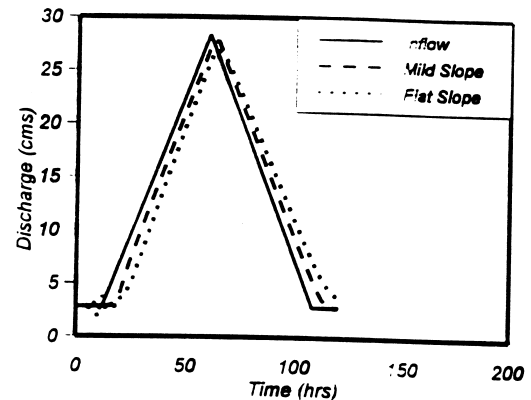


Figure 5. Discharge hydrographs on R3.

occurs in 64 hours. The peak discharge propagates through river R3 in 4 hours and its attenuation is 0.6 cms. When modeling river R3 as a dynamic tributary with a flat slope, a maximum discharge at the reference section of 34.6 cms occurs in 66 hours. The maximum discharge in R3 attenuates 1.4 cms in 6 hours. The difference in the peak discharge at the reference section is due to the additional attenuation that occurred in the flat sloping tributary. The relaxation technique requires about two iterations at each time step during the solution.

### Conclusion

In the NWS FLDWAV model, the relaxation technique for simulating unsteady flows in a dendritic (tree-type) network of channels with 1st-order tributaries is extended to handle nth order tributaries. By specifying the number of the river into which the tributary flows, the current relaxation algorithm can be used without modification. A new algorithm is used to compute initial conditions and to allow the flexibility of adding tributaries to the system in any order without affecting the numbering scheme of the original river system.

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